Decomposing Complete 3-uniform Hypergraph $K_{58}^{(3)}$ into 7-cycles with Computer Aid

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Abstract. On the basic of the definition of Hamiltonian cycle defined by Katona-Kierstead and Jianfang Wang independently. Some domestic and foreign researchers study the decomposition of complete 3-uniform hypergraph $K_n^{(3)}$ into Hamiltonian cycles and not Hamiltonian cycles. Especially , Bailey Stevens using Clique - finding the decomposition of $K_n^{(3)}$ into Hamiltonian cycles for $K_7^{(3)}$, $K_8^{(3)}$. Meszka-Rosa showed that Hamiltonian decompositions of $K_n^{(3)}$ for all admissible $n \le 32$. Meszka-Rosa proved that a decomposition of $K_n^{(3)}$ into 5-cycles has been presented for all admissible $n \le 17$, and for all $n = 4^m + 1$, m is a positive integer. In general, the existence of a decomposition into $l(\ge 5)$-cycles remains open. The authors have given the decomposition of $K_n^{(3)}$ into 7-cycles for $n \in \{7,8,14,16,22,23,29,37,43,44,50\}$ and has showed if $K_n^{(3)}$ can be decomposition into 7-cycles, then $K_{7n}^{(3)}$ can be decomposition into 7-cycles. In this paper, a decomposition of $K_{58}^{(3)}$ into 7-cycles is proved using the method of edge-partition and cycle sequence proposed by Jirimutu.

Introduction

The definition of hypergraph in Hamilton was first put forward by Berge in 1970. The decomposition of hypergraph Berge Hamiltonian cycle has been completely solved by Verrall in 1994.It is also worth to mention a definition of a cycle by Berge. Many papers studied the two different definitions of a Hamiltonian cycle in [1-2], which are due to Katona and Kierstead, Wang and Lee respectively. In fact, the two different definitions of Hamiltonian chain and Hamiltonian cycle are the same. Some researchers studied the decomposition of complete 3-uniform hypergraph $K_n^{(3)}$ into Hamiltonian cycles and not Hamiltonian cycles in [2-9]. Especially , Bailey Stevens [3] used clique-finding the decomposition of $K_n^{(3)}$ into Hamiltonian cycles for $K_7^{(3)}, K_8^{(3)}$ and Meszka-Rosa [4] showed that Hamiltonian decompositions of $K_n^{(3)}$ for all admissible $n \le 32$. Huo [10] obtained some results for all admissible $32 \le n \le 46$ and $n \ne 43$ using the method of edge-partition and cycle sequence. The problem of decomposing the complete 3-uniform hypergraph into 5-cycles and $l(>5)$ are open. Meszka-Rosa [4] proved that a decomposition of $K_n^{(3)}$ into 5-cycles has been presented for all admissible $n \le 17$, and for all $n = 4^m + 1$, m is a positive integer. Meszka-Rosa [4] have introduced a necessary condition for the existence of 5-cycles such a decomposition is that $n \equiv 1,2,5,7,10, 11 \pmod{15}$. Li [11] find a decomposition of $K_n^{(3)}$ into 5-cycles for $n \in \{5,7,10,11,16,17,20,22,26\}$ and has showed if $K_n^{(3)}$ can be decomposition into 5-cycles, then

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$K_{5n}^{(3)}$ can be decomposed into 5-cycles. Li and Hong in [11][12] find a decomposition of $K_n^{(3)}$ into 7-cycles for $n \in \{7,8,14,16,22,23,29,37,43,44,50\}$ and Li [13] has showed if $K_n^{(3)}$ can be decomposition into 7-cycles, then $K_{7n}^{(3)}$ can be decomposition into 7-cycles. In this paper we find $K_{58}^{(3)}$ decomposed into 7- cycles.

Preliminaries

A hypergraph $H=(V,E)$ consists of a finite set V of vertices with a family E of subsets of V, called hyperedges (or simply edges). If each (hyper)edge has size k, we say that H is a k-uniform hypergraph. In particular, the complete k-uniform hypergraph on n vertices has all k-subsets of $Z_n=\{0,1,\cdots,n-1\}$ as edges, denoted it by $K_n^{(k)}$. The set of (hyper)edges of $K_n^{(3)}$ is denoted it by $\varepsilon\left(K_n^{(3)}\right)$.

Definition 1. Let $H=(V,E)$ be a k-uniform hypergraph. An l-*cycle* in H is a cyclic ordering k of $(v_0,v_1,\cdots,v_{l-1})$ where $3\le k\le l-1$ such that each consecutive k -tuple of vertices is an edge of H.

Definition 2. An l-*cycle decomposition* of H is a partition of the set of (hyper)edges of H into mutually-edge-disjoint l-cycles.

We introduce the method of edge-partition and cycle-model from in [5].

Definition 3.[4] Let $T=\{a,b,c\}$ be a triple of distinct elements of Z_n. Then its *difference pattern* $\pi(T)$, is the equivalence class of ordered triples containing cyclic rotations of $(b-a,c-b,a-c)$ and $(c-a,b-c,a-b)$ (where the differences are taken modulo n).

Clearly, the three differences sum to zero. Therefore if we know that the first two differences are x and y, then the third is $n-x-y$. Omit the third number, we obtain get a difference pair. Using edge-partition of $K_n^{(3)}$ as in paper [5], all difference pairs of hypergraphs $K_n^{(3)}$ may be obtained. Let Z be the set of integers, n be a fixed positive integer, and $Z_n=\{0,1,\cdots,n-1\}$. Let

$$D_{all}(n)=\{\ (k_1,k_2)\mid 1\le k_1,k_2\le n-1,\ and\ k_1+k_2\ne n\ \}$$

$$D(n)=D_e(n)\cup D_l(n)\cup D_m(n)$$

where

$$D_e(n)=\left\{\ (k_1,k_2)\in D_{all}(n)\ \middle|\ k_1=k_2=k,\ \text{且}\ 1\le k<\frac{n}{2}\ \right\}$$

$$D_l(n)=\left\{\ (k_1,k_2)\in D_{all}(n)\ \middle|\ 1\le k_1<k_2<\frac{n-k_1}{2}\ \right\}$$

$$D_m(n)=\{\ (k_2,k_1)\in D_{all}(n)\ \mid\ (k_1,k_2)\in D_l(n)\ \}.$$

Given a difference pair $(k_1,k_2)\in D_{all}(n)$ and an integer $m\in Z_n$, define a subhypergraph of $K_n^{(3)}$ generated by (k_1,k_2) as follows:

$$E(m;k_1,k_2)\equiv\{\ m,\ m+k_1,\ m+k_1+k_2\ \}\pmod n$$

Introduce the notation by

$$H(k_1,k_2)=\{E(m;k_1,k_2)\mid m\in Z_n\}$$

where the addition is performed modulo n.

Now we repeat some of the results from to make the paper self-contained.

Lemma 1.[5] Let (k_1,k_2) and (k_1',k_2') be arbitrary two distinct difference pairs in $D_{all}(n)$, we have either $H(k_1,k_2)\cap H(k_1',k_2')=\varnothing$ or $H(k_1,k_2)=H(k_1',k_2')$, and a necessary and sufficient condition for the second equation is

$$(k_1,k_2)\equiv\begin{cases}(k_1',\ k_2') & or\\ (k_1'+k_2',-k_2') & or\\ (-k_1',\ k_1'+k_2') & or\\ (k_2',-k_1'-k_2') & or\\ (-k_1'-k_2',\ k_1') & or\\ (-k_2',-k_1') & \end{cases}\quad (\text{mod } n).$$

Definition 4[5]. Let (k_1,k_2) and (k_1',k_2') be arbitrary two distinct difference pairs in $D_{all}(n)$. We say (k_1,k_2) and (k_1',k_2') are equivalent if $H(k_1,k_2)=H(k_1',k_2')$. This is denoted by $(k_1,\ k_2)\sim(k_1',k_2')$.

Lemma 2[5]**.** (Edge-partition of $K_n^{(3)}$)For any $K_n^{(3)}$,

$$\varepsilon\left(K_n^{(3)}\right)=\bigcup_{(k_1,k_2)\in D(n)}H(k_1,k_2),$$

where $(k_1,k_2)\in D(n)$. If $k_1\neq k_2$, for convenience, we use (k_1,k_2) denote (k_1,k_2) and (k_2,k_1).

Definition 5. Let n be a positive integer, for any $0\le i,j\le l-1$, $(k_i,k_{i+1})\in D_{all}(n)$, (k_i,k_{i+1}) and (k_j,k_{j+1}) are inequitable when $i\neq j$, obtain that $(k_0,\ k_1,\ \cdots,\ k_{l-1})$ be a sequence on $D_{all}(n)$.The sequence $(k_0,\ k_1,\ \cdots,\ k_{l-1})$ induces the cycle sequence

$$(r_0,\ r_1,\ \cdots,\ r_{l-1}) \tag{1}$$

Sequence Eq.1 satisfies the following two conditions:

$a)$. $r_0=0$, $\sum_{i=0}^{j}k_i\equiv r_j\ (\text{mod } n)$, $r_l=r_0=0$.

$b)$. For any $i,j\ (i\neq j)$, $r_i\neq r_j$.

Then $(r_0,\ r_1,\ \cdots,\ r_{l-1})$ is an l-cycle, denoted by $C_l=(r_0,\ r_1,\ \cdots,\ r_{l-1})$, called *base cycle*. According to the definition of difference pattern $\pi(T)$, obviously, we obtained the set of l-cycles $\{C_l+i\mid i\in Z_n\}$, where $C_l+i=(r_0+i,\ r_1+i,\ \cdots,\ r_{l-1}+i)(\text{mod}\, n)$. In particular, if $l=n$, then $(r_0,\ r_1,\ \cdots,\ r_{n-1})$ is a base Hamiltonian cycle, denoted by $C_n=(r_0,\ r_1,\ \cdots,\ r_{n-1})$.

Lemma 3. Let n be a positive integer, for any $0\le i,j\le l-1$, $(k_i,k_{i+1})\in D_{all}(n)$, (k_i,k_{i+1}) and

(k_j, k_{j+1}) are inequitable when $i \neq j$, if $(k_0,\ k_1,\ \cdots,\ k_{l-1})$ be a sequence on $D_{all}(n)$, then

$$H(k_0,\ k_1,\ \cdots,\ k_{l-1}) = \bigcup_{i=0}^{l-1} H(k_i,\ k_{i+1}),$$

where $k_l = k_0$.

Decomposing $K_{58}^{(3)}$ Into 7-cycles

Theorem 1. $K_{58}^{(3)}$ can be decomposed into 7-cycles.

Proof. We can decompose the edges of $K_{58}^{(3)}$ into 4408 7-cycles produced by 76-base 7-cycles as follows: we have since $\left|\varepsilon\left(K_{58}^{(3)}\right)\right| = \binom{58}{3} = 30856$ edges and 7|30856, we have

$$\begin{aligned} D_{all}(58) = \{&(1,1),(2,2),(3,3),(4,4),(5,5),\ (6,6),(7,7),(8,8),(9,9),\ (10,10),(11,11),(12,12),\\ &(13,13),(14,14),(15,15),(16,16),(17,17),(18,18),(19,19),(20,20),(21,21),(22,22),\\ &(23,23),(24,24),(25,25),\ (26,26),(27,27),(28,28),\ (1,2),(1,3),(1,4),(1,5),(1,6),\\ &(1,7),(1,8),\ (1,9),(1,10),\ (1,11),(1,12),(1,13),\ (1,14),(1,15),\ (1,16),(1,17),(1,18),\\ &(1,19),(1,20),\ (1,21),(1,22),(1,23),(1,24),(1,25),(1,26),(1,27),(1,28),(2,3),\ (2,4),\\ &(2,5),(2,6),\ (2,7),(2,8),(2,9),(2,10),(2,11),(2,12),\ (2,13),(2,14),(2,15),(2,16),\\ &(2,17),(2,18),\ (2,19),(2,20),(2,21),\ (2,22),(2,23),\ (2,24),(2,25),(2,26),(2,27),\\ &(3,4),(3,5),\ (3,6),(3,7),(3,8),(3,9),\ (3,10),(3,11),\ (3,12),\ (3,13),(3,14),(3,15),\\ &(3,16),(3,17),(3,18),\ (3,19),(3,20),(3,21),(3,22),\ (3,23),(3,24),\ (3,25),(3,26),\\ &(3,27),(4,5),\ (4,6),(4,7),(4,8),(4,9),(4,10),(4,11),\ (4,12),(4,13),(4,14),(4,15),\\ &(4,16),(4,17),(4,18),(4,19),\ (4,20),(4,21),(4,22),(4,23),\ (4,24),(4,25),(4,26),\\ &(5,6),\ (5,7),\ (5,8),\ (5,9),\ (5,10),(5,11),\ (5,12),(5,13),\ (5,14),(5,15),\ (5,16),\\ &(5,17),\ (5,18),(5,19),\ (5,20),(5,21),\ (5,22),(5,23),(5,24),(5,25),\ (5,26),(6,7),\\ &(6,8),(6,9),(6,10),(6,11),(6,12),(6,13),(6,14),(6,15),(6,16),(6,17),(6,18),(6,19),\\ &(6,20),(6,21),\ (6,22),(6,23),(6,24),\ (6,25),(7,8),(7,9),(7,10),\ (7,11),(7,12),\\ &(7,13),(7,14),\ (7,15),(7,16),\ (7,17),(7,18),\ (7,19),(7,20),(7,21),(7,22),(7,23),\\ &(7,24),(7,25),\ (8,9),(8,10),\ (8,11),(8,12),\ (8,13),\ (8,14),(8,15),\ (8,16),(8,17),\\ &(8,18),(8,19),(8,20),(8,21),(8,22),(8,23),(8,24),(9,10),\ (9,11),(9,12),\ (9,13),\\ &(9,14),(9,15),(9,16),(9,17),(9,18),\ (9,19),(9,20),(9,21),(9,22),(9,23),(9,24),\\ &(10,11),(10,12),(10,13),(10,14),(10,15),(10,16),(10,17),(10,18),(10,19),(10,20),\\ &(10,21),(10,21),(10,22),(10,23),(11,12),(11,13),(11,14),(11,15),(11,16),(11,17),\\ &(11,18),(11,19),(11,20),(11,21),(11,22),(11,23),(12,13),(12,14),(12,15),(12,16),\\ &(12,17),(12,18),(12,19),(12,20),(12,21),(12,22),(13,14),(13,15),(13,16),(13,17),\\ &(13,18),(13,19),(13,20),(13,21),(13,22),(14,15),(14,16),(14,17),(14,18),(14,19),\\ &(14,20),(14,21),(15,16),(15,17),(15,18),(15,19),(15,20),(15,21),(16,17),\ (8,18),\\ &(8,19),(16,17),(16,18),(16,19),(16,20),(17,18),(17,19),(17,20),(18,19).\} \end{aligned}$$

Now, we need to find the decomposition of $K_{58}^{(3)}$. On $D(58)$, according to Definition 5, we obtain 76 sequences as follows:

$(1)(1,4,56,5,52,5,51)$, $(2)(1,7,54,56,8,2,46)$, $(3)(1,9,51,4,1,10,40)$,
$(4)(1,12,48,2,9,1,43)$ $(5)(1,13,47,3,2,12,38)$ $(6)(1,15,45,3,3,12,37)$
$(7)(1,16,44,3,5,8,39)$ $(8)(1,21,39,2,15,3,35)$ $(9)(1,23,37,3,6,13,33)$
$(10)(1,25,35,3,7,14,31)$ $(11)(1,27,33,3,9,14,29)$ $(12)(1,29,31,3,13,12,27)$
$(13)(1,31,29,3,1,1,2)$ $(14)(1,3,1,49,14,13,25)$ $(15)(1,33,27,3,15,15,22)$
$(16)(1,34,26,2,16,16,21)$ $(17)(1,37,23,2,18,16,19)$ $(18)(1,39,21,3,16,20,16)$
$(19)(1,40,20,2,20,18,15)$ $(20)(1,44,16,2,22,24,7)$ $(21)(1,45,15,2,24,19,10)$
$(22)(1,46,14,3,17,29,6)$ $(23)(2,26,35,54,6,29,22)$ $(24)(2,28,33,54,8,32,17)$
$(25)(2,45,16,54,9,39,9)$ $(26)(2,49,12,50,50,6,5)$ $(27)(3,19,43,52,9,20,28)$
$(28)(3,21,41,52,10,21,26)$ $(29)(3,30,32,52,12,21,24)$ $(30)(3,32,30,52,13,22,22,)$
$(31)(3,36,26,52,14,26,17)$ $(32)(3,37,25,52,15,30,12)$ $(33)(3,39,23,52,16,31,10)$
$(34)(3,44,18,52,17,32,8)$ $(35)(3,48,14,52,18,34,5)$ $(36)(4,6,14,36,4,19,33)$
$(37)(4,7,4,45,4,20,32)$ $(38)(4,10,53,50,12,5,40)$ $(39)(4,11,52,51,12,14,30)$
$(40)(4,12,51,50,17,13,27)$ $(41)(4,13,4,38,4,29,24)$ $(42)(4,31,32,50,19,21,17)$
$(43)(4,34,33,46,19,27,11)$ $(44)(4,36,27,40,43,19,5)$ $(45)(5,9,5,42,5,13,37)$
$(46)(5,10,5,39,5,21,31)$ $(47)(5,12,8,34,7,15,35)$ $(48)(5,15,50,48,16,7,33)$
$(49)(5,17,48,47,17,10,30)$ $(50)(5,24,42,48,17,12,26)$ $(51)(5,25,40,47,18,18,21)$
$(52)(5,28,36,48,20,14,23)$ $(53)(5,33,31,48,22,17,18)$ $(54)(5,36,30,34,38,15,16)$
$(55)(6,16,49,46,22,19,16)$ $(56)(6,25,41,45,20,23,14)$ $(57)(6,30,35,46,23,25,9)$
$(58)(6,31,34,46,46,51,18)$ $(59)(6,37,28,47,33,10,13)$ $(60)(7,7,13,32,7,20,30)$
$(61)(7,16,51,43,23,11,23)$ $(62)(7,22,45,43,24,16,17)$ $(63)(7,25,42,44,22,15,19)$
$(64)(7,26,43,44,21,13,20)$ $(65)(8,19,8,24,10,19,28)$ $(66)(8,21,47,42,25,8,23)$
$(67)(8,29,42,41,43,49,20)$ $(68)(8,37,41,31,49,49,17)$ $(69)(9,23,10,18,27,16,13)$
$(70)(9,26,46,42,26,14,11)$ $(71)(9,28,49,40,29,49,28)$ $(72)(9,35,41,34,11,17,27)$
$(73)(10,14,28,20,19,11,14)$ $(74)(10,35,43,25,14,25,22)$ $(75)(11,12,18,12,31,12,20)$
$(76)(11,26,13,11,32,45,36)$

Let D be a collection of the 76 sequences above. Thus they correspond to 76-base 7-cycles:

$C_{7(1)}=(0,1,2,4,5,8,9)$ $C_{7(2)}=(0,1,5,3,8,2,7)$ $C_{7(3)}=(0,1,8,4,2,10,12)$
$C_{7(4)}=(0,1,10,3,7,8,18)$ $C_{7(5)}=(0,1,13,3,5,14,15)$ $C_{7(6)}=(0,1,14,3,6,8,20)$
$C_{7(7)}=(0,1,16,3,6,9,21)$ $C_{7(8)}=(0,1,17,3,6,11,19)$ $C_{7(9)}=(0,1,22,3,5,20,23)$
$C_{7(10)}=(0,1,24,3,6,12,25)$ $C_{7(11)}=(0,1,26,3,6,13,27)$ $C_{7(12)}=(0,1,28,3,6,15,29)$
$C_{7(13)}=(0,1,30,3,6,19,31)$ $C_{7(14)}=(0,1,32,3,6,20,33)$ $C_{7(15)}=(0,1,34,3,6,21,36)$
$C_{7(16)}=(0,1,35,3,5,21,37)$ $C_{7(17)}=(0,1,38,3,5,23,39)$ $C_{7(18)}=(0,1,40,3,6,22,42)$
$C_{7(19)}=(0,1,41,3,5,25,43)$ $C_{7(20)}=(0,1,45,3,5,27,51)$ $C_{7(21)}=(0,1,46,3,5,29,48)$
$C_{7(22)}=(0,1,47,3,6,23,52)$ $C_{7(23)}=(0,2,28,5,1,7,36)$ $C_{7(24)}=(0,2,30,5,1,9,41)$
$C_{7(25)}=(0,2,47,5,1,10,49)$ $C_{7(26)}=(0,2,51,5,55,47,53)$ $C_{7(27)}=(0,3,22,7,1,10,30)$
$C_{7(28)}=(0,3,24,7,1,11,32)$ $C_{7(29)}=(0,3,33,7,1,13,34)$ $C_{7(30)}=(0,3,35,7,1,14,36)$
$C_{7(31)}=(0,3,39,7,1,15,41)$ $C_{7(32)}=(0,3,40,7,1,16,46)$ $C_{7(33)}=(0,3,42,7,1,17,48)$
$C_{7(34)}=(0,3,47,7,1,18,50)$ $C_{7(35)}=(0,3,51,7,1,19,53)$ $C_{7(36)}=(0,4,10,24,2,6,25)$
$C_{7(37)}=(0,4,11,15,2,6,26)$ $C_{7(38)}=(0,4,14,9,1,13,18)$ $C_{7(39)}=(0,4,15,9,2,14,28)$
$C_{7(40)}=(0,4,16,9,1,18,31)$ $C_{7(41)}=(0,4,16,9,1,18,31)$ $C_{7(42)}=(0,4,35,9,1,20,41)$
$C_{7(43)}=(0,4,38,13,1,20,47)$ $C_{7(44)}=(0,4,40,9,49,34,53)$ $C_{7(45)}=(0,5,14,19,3,8,21)$
$C_{7(46)}=(0,5,15,20,1,6,27)$ $C_{7(47)}=(0,5,17,25,1,8,23)$ $C_{7(48)}=(0,5,20,12,2,18,25)$
$C_{7(49)}=(0,5,22,12,1,18,28)$ $C_{7(50)}=(0,5,29,13,3,20,32)$ $C_{7(51)}=(0,5,30,12,1,19,37)$
$C_{7(52)}=(0,5,33,11,1,21,35)$ $C_{7(53)}=(0,5,38,11,1,23,40)$ $C_{7(54)}=(0,5,41,13,47,27,42)$
$C_{7(55)}=(0,6,22,13,1,23,42)$ $C_{7(56)}=(0,6,31,14,1,21,44)$ $C_{7(57)}=(0,6,36,13,1,24,49)$
$C_{7(58)}=(0,6,37,13,1,47,40)$ $C_{7(59)}=(0,6,43,13,2,35,45)$ $C_{7(60)}=(0,7,14,27,1,8,28)$
$C_{7(61)}=(0,7,23,16,1,24,35)$ $C_{7(62)}=(0,7,29,16,1,25,41)$ $C_{7(63)}=(0,7,32,16,2,24,39)$
$C_{7(64)}=(0,7,33,18,4,25,38)$ $C_{7(65)}=(0,8,27,35,1,11,30)$ $C_{7(66)}=(0,8,29,18,2,27,35)$
$C_{7(67)}=(0,8,37,21,4,47,38)$ $C_{7(68)}=(0,8,45,28,1,50,41)$ $C_{7(69)}=(0,9,32,42,2,29,45)$
$C_{7(70)}=(0,9,35,23,7,33,47)$ $C_{7(71)}=(0,9,37,28,10,39,30)$ $C_{7(72)}=(0,9,44,27,3,14,31)$
$C_{7(73)}=(0,10,24,52,14,33,44)$ $C_{7(74)}=(0,10,45,30,55,11,36)$ $C_{7(75)}=(0,11,23,41,53,26,38)$
$C_{7(76)}=(0,11,37,50,3,35,22)$

By the method of edge-partition, we obtain the decomposition of $K_{58}^{(3)}$ into 4408 7-cycles, that is

$$\varepsilon\left(K_{58}^{(3)}\right)=\bigcup_{(k_1,k_2)\in D(58)} H(k_1,k_2)=\bigcup_{(k_{i0},k_{i1}\cdots k_{i6})\in D_{all}(58)} H(k_{i0},k_{i1},\cdots,k_{i6})$$
$$=\bigcup_{i=1}^{76}\left\{C_{7(i)}+j,\ j\in Z_{58}\right\}$$

where $C_{7(i)} = (r_{i0},\ r_{i1},\ \cdots,\ r_{i6})$, $C_{7(i)} + j = \{r_{i0} + j,\ r_{i1} + j,\ \cdots, r_{i6} + j\} (\bmod 58)$.

Hence we obtain the decomposition of $K_{58}^{(3)}$ into 4408 7-cycles.

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